

## Opgave 1

1)

$$\begin{aligned} p(-2) + p(-1) + p(0) + p(1) + p(2) &= c(4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}) \\ &= c(7\frac{3}{4}) = c \cdot \frac{31}{4} = 1 \end{aligned}$$

$$\text{så } c = \frac{4}{31}$$

2)

$$E(X) = \frac{4}{31}(-2 \cdot 4 + (-1) \cdot 2 + 0 + \frac{1}{2} + 2 \cdot \frac{1}{4}) = \frac{4}{31}(-1) = -\frac{36}{31}$$

3)  $Y = |X|$ . Lad  $q$  være ss.fkt for  $Y$ . Så er

$$\begin{aligned} q(0) &= p(0) = \frac{4}{31} \\ q(1) &= p(-1) + p(1) = \frac{4}{31}(2 + \frac{1}{2}) = \frac{4}{31} \cdot \frac{5}{2} = \frac{10}{31} \\ q(2) &= p(-2) + p(2) = \frac{4}{31}(4 + \frac{1}{4}) = \frac{4}{31} \cdot \frac{17}{4} = \frac{17}{31} \\ q(y) &= 0 \text{ for } y \neq 0, 1, 2. \end{aligned}$$

Eller mere kompakt

$$q(y) = \begin{cases} \frac{4}{31} & y = 0 \\ \frac{4}{31} \left( (\frac{1}{2})^{-y} + (\frac{1}{2})^y \right) & y = 1, 2 \\ 0 & \text{ellers} \end{cases}$$

4)

$$P(X \geq -1) = 1 - P(X = -2) = 1 - \frac{4}{31} \cdot 4 = \frac{15}{31}$$

$$P(Y = 1 \text{ og } X \geq -1) = P(X = -1 \vee X = 1) = \frac{4}{31} \left(2 + \frac{1}{2}\right) = \frac{10}{31}$$

$$P(Y = 2 \text{ og } X \geq -1) = P(X = 2) = \frac{4}{31} \cdot \frac{1}{4} = \frac{1}{31}$$

så

$$P(Y \geq 1 \text{ og } X \geq -1) = \frac{11}{31}$$

dvs.

$$P(Y \geq 1 | X \geq -1) = \frac{11}{31} / \frac{15}{31} = \frac{11}{15}.$$